# Numerical values for Nuclear and Quantum Physics

Final version

$\sigma.\beta.$

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## 1 Fundamental constants

$c = 2.998\ 10^{8}\ m/s$
$h = 6.626\ 10^{-34}\ Js$
$\hbar = 1.055\ 10^{-34} Js = 6.582\ 10^{-22}$ MeV $s$
$\hbar c = 197.327$ MeV $fm$
$m_p = 938.272$ MeV
$m_n = 939.565$ MeV
$\Delta m = m_n - m_p = 1.293$ MeV
$m_e = 0.511$ MeV

## 2 Atomic constants

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = 7.297\ 10^{-3} \approx 1/137$$
$$\frac{e^2}{4\pi\varepsilon_0} = 1.44 \text{ MeV } fm$$
$$r_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_e c^2} = 2.818\ fm$$
$$r_{Bohr} = 4\pi\varepsilon_0\frac{\hbar^2}{m_e e^2} = \frac{r_e}{\alpha^2} = 0.529\ \text{Å}$$
$$Ry = \frac{1}{(4\pi\varepsilon_0)^2}\frac{m_e e^4}{2\hbar^2} = m_e c^2\alpha^2/2 = 13.606 \text{ eV}$$
$$\mu_{Bohr} = \frac{e\hbar}{2m_e} = 5.788\ 10^{-11} \text{ MeV}/T$$
$$\mu_N = \mu_{nuclear} = \frac{e\hbar}{2m_p} = 3.152\ 10^{-14} \text{ MeV}/T$$

## 3 Liquid drop model of the nucleus

Binding energy

$$B = a_r A - a_s A^{2/3} - a_c Z^2/A^{1/3} - a_{sym}|N-Z|^2/A + \delta(A)$$

with

$$\begin{aligned} a_r &= 15.69\ MeV \\ a_s &= 16.56\ MeV \\ a_c &= 0.717\ MeV \\ a_{sym} &= 28 \div 30\ MeV \\ a_p &\approx 34\ MeV \end{aligned}$$

where the pairing effects is given by:

$$\delta(A) = \begin{cases} a_p A^{-3/4} & \text{pari-pari} \\ 0 & \text{disp-pari} \\ -a_p A^{-3/4} & \text{disp-disp} \end{cases}$$

Please note that there are different definitions for the power of $A$ in $\delta(A)$ in books, this should be the one accepted in our classes.

# 4 Deutone aka Deuteron

Nucleus of deuterium

$$\begin{aligned} &J^P = 1^+ \\ &E = -B = -2.225\ MeV \\ &R_d{}^{rms} = 2.1\ fm \\ &Q_d = 0.00282\ \text{barn} \\ &Q_d/(4\pi R_d^2) \approx 1/196 \sim 0.5\% \end{aligned}$$

It is the mixing of $^3S_1$ and $^3D_1$ (the latter accounting for roughly 3.9%).

# 5 Magnetic dipoles

Orbital:

$$\mu_l = g_l \frac{e\hbar}{2m}\frac{l}{\hbar}$$

Spin:

$$\mu_s = g_s \frac{e\hbar}{2m}\frac{s}{\hbar}$$

$$\mu_{tot} = \mu_l + \mu_s$$

In the case of neutron and proton:

$$\mu_N^p = \frac{e\hbar}{2m_p} \approx \frac{e\hbar}{2m_n} = \mu_N^n$$

Orbital gyromagnetic factors:

$$\begin{aligned} g_l^n &= 0 \rightarrow \mu_l^n = 0 \\ g_l^p &= 1 \rightarrow \mu_l^p = 1 \cdot \mu_N \frac{l}{\hbar} \\ g_l^e &= 1 \rightarrow \mu_l^e = 1 \cdot \mu_{Bohr} \frac{l}{\hbar} \end{aligned}$$

Spin g-factors (nucleons and $e^-$ have $s = \hbar/2$)

$$\begin{aligned} g_s^n &= -3.826 && \rightarrow && \mu_s^n = (g_s^n/2)\mu_N = 2.79\mu_N \\ g_s^p &= 5.586 && \rightarrow && \mu_s^p = (g_s^p/2)\mu_N = -1.91\mu_N \\ g_s^e &= -2.002 && \rightarrow && \mu_s^e = (g_s^e/2)\mu_{Bohr} = -\mu_{Bohr} \end{aligned}$$

# 6 Scattering between nucleons

Scattering lenght

$$a = -\lim_{k\to 0} \frac{\tan\delta_0}{k}$$

## 6.1 Proton-neutron

$$a_{sing} = (-23.55 \pm 0.12)\ fm \ < 0 \rightarrow \text{no bound state}$$
$$a_{trip} = (5.35 \pm 0.06)\ fm \ > 0 \rightarrow \text{weak bound state}$$

Fitting experimental data with Bethe's formula for the effective radius $r_0$:

$$k\ \text{cotan}\,\delta_0 = -\frac{1}{a} + \frac{1}{2} r_0 k^2 + o(k^4)$$

Singlet:

$$a_s^* = (-23.715 \pm 0.015)\ fm$$
$$r_0^s = (2.73 \pm 0.03)\ fm$$

Triplet:

$$a_t^* = (5.423 \pm 0.05)\ fm$$
$$r_0^t = (1.748 \pm 0.006)\ fm$$

## 6.2 Proton-proton

Scattering due to both the electric and nuclear interactions:

$$\text{Nuclear+Electric:} \begin{cases} a^{pp\prime} = (-7.82 \pm 0.01)\ fm \\ r_o^{pp\prime} = (2.79 \pm 0.02)\ fm \end{cases}$$

If the electric interaction is removed:

$$\text{Nuclear only:} \begin{cases} a^{pp} = (-17.1 \pm 0.2)\ fm \\ r_o^{pp} = (2.84 \pm 0.03)\ fm \end{cases}$$

## 6.3 Neutron-neutron

$$\begin{cases} a^{nn} = (-16.6 \pm 0.5)\ fm \\ r_o^{pp} = (2.66 \pm 0.15)\ fm \end{cases}$$

# 7 Mean life for common particles

## 7.1 Baryons

$\tau_p \sim 10^{29}$ yrs
$\tau_n = 880\ s \sim 15$ min

## 7.2 Leptons

$m_{\mu\pm} = 105.658\ MeV$
$\tau_{\mu\pm} = 2.2\ \mu s$
$m_{\tau\pm} = 1776\ MeV$
$\tau_{\tau\pm} = 290\ 10^{-15}\ s$

### 7.3 Mesons

$m_{\pi\pm} = 140\ MeV$
$\tau_{\pi\pm} = 0.26\ ns$
$m_{\pi 0} = 134.9\ MeV$
$\tau_{\pi 0} = 8.5\ 10^{-17} s$
$m_{K\pm} = 494\ MeV$

## 8 Useful facts on accelerators

### 8.1 Luminosity

| *Facility* | *Type* | $\mathcal{L}$ | *Energy* |
|---|---|---|---|
| $Da\phi ne\ (Frascati)$ | $e^+e^-$ | $500\ \mu b^{-1} s^{-1}$ | 1 GeV |
| $Tevatron\ (Stanford)$ | $p\bar{p}$ | $25\ \mu b^{-1} s^{-1}$ | 2 $TeV$ |
| $LHC\ (Geneva)$ | $pp$ | $10^6\ b^{-1} s^{-1}$ in 2010 | 14 $TeV$ |

### 8.2 CERN's acceleration chain

| *Accelerator* | *Exit Energy per beam* |
|---|---|
| $LINAC$ 2 | 50 $MeV$ |
| $PSB$ | 1.4 $GeV$ |
| $PS$ | 25 $GeV$ |
| $SPS$ | 450 $GeV$ |
| $LHC$ | 8 $TeV$ |

## 9 Energy loss for a charge moving in matter

In Bethe-Bloch region, the minimum ionization point is in $\beta\gamma \sim 3$ which gives a stopping power of $dE/dx \sim -(1.5 \div 2) MeV/cm$ for unitary density.
Cerenkov visible radiation emitted by a particle at $\beta = 1$ in glass-like material ($n \sim 1.3$) is roughly 1 $k$eV$/cm$.

## 10 Radiation vs matter

- Ultra low energy: photoelectric effect usually for $E_\gamma \sim 10 \div 100$ eV;
- Low energy: Rayleigh elastic scattering, for photons whose $\lambda >> d$ dimension of target particles (in gas or liquid). Typical values for $d \sim 1$Å are $E_\gamma \sim 1 \div 10$ eV (visible light) or higher;
- Mid-low energy: Thomson elastic scattering ($h\nu << mc^2$) on a target mass $m$. If using electrons, $E_\gamma << 500\ k$eV, i.e. Thomson should dominate till $E_\gamma \sim 50\ k$eV;
- High energy: Compton anelastic scattering on a weakly bound electron, the photon having $\sim 5\ 10^2\ k$eV;
- Ultra high energy: $e^+e^-$ pair production, for $\gamma$ with $E_\gamma > 2m_e \sim 1\ MeV$ on static nuclei or $E_\gamma > 4m_e$ on electrons in rest.

# 11 Electrons in matter: Moeller scattering

All the machinery for energy loss in matter depends on the hypotesis that the projectile has $m << M$ of the target. When considering a target mass comparable to the electron's one, Bethe-Bloch and Bremsstrahlung do not work anymore. There still exists a critical energy $E_c$ such that for $e^-$ above this value, radiation loss prevail on ionization. For a relativistic electron, we have in different materials:

| *Material* | $\beta\gamma = \gamma$ | $E_c$ |
|---|---|---|
| *Pb* | 20 | 10 $MeV$ |
| *Air* | 200 | 100 $MeV$ |

# 12 Radiation lenght

When electrons lose energy by Bremsstrahlung or pair production, there exists a characteristic lenght over which the $e^-$ reduces its energy to $(1/e)E_0$, corresponding to 7/9 of the path for creating a couple $e^-e^+$.

| *Material* | $X_0$ $(cm)$ | $X'_0$ $(g/cm^2)$ |
|---|---|---|
| *Pb* | 0.56 | 6.37 |
| *Fe* | 1.76 | 13.84 |
| *Ur* | 0.32 | 6.00 |
| *Air* | 30000 | 36.7 |

# 13 Strenght of interactions

Some constants are still running, but we'll catch'em one day.

$$\alpha_{EM} \sim 1/137$$
$$\alpha_{strong} \sim 1$$
$$\alpha_{weak} \sim 10^{-5} \text{ for } m_p$$
$$\alpha_{gravity} \sim 10^{-39}$$

Most of the values above are taken from the PDG booklet or the notes of our classes of Fisica 3. No responsibility is taken for possible mistakes. Corrections and suggestions are welcome.