TRANSIZIONI DI FASE QUANTISTICHE

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Le transizioni di fase quantistiche mostrano cambiamenti qualitativi nelle proprietá dello stato fondamentale in sistemi a molti corpi. A differenza delle transizioni di fase classiche, che sono generalmente causate da fluttuazioni termiche (per esempio la transizione liquido-gas), le transizioni quantistiche hanno origine dalle fluttuazioni quantistiche a temperatura zero.

Attivitá di ricerca degli ultimi anni

lavori in collaborazione con Giacomo Ceccarelli, Massimo Campostrini, Francesco Delfino, Jacopo Nespolo, Andrea Pelissetto, Subir Sachdev, ...

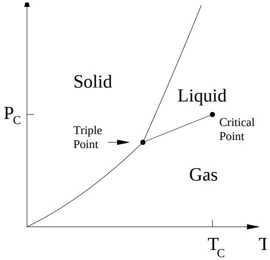
Phase transitions and Critical phenomena are observed in many physical systems

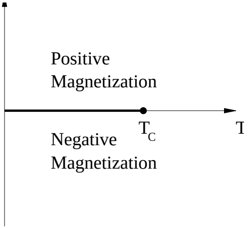
There are two broad classes of phase transitions:

first order $\rightarrow$ discontinuity in thermodynamic quantities, such as the energy density

continuous $\rightarrow$ nonanalytic behavior due to a diverging length scale characterizing the physical correlations

Examples of **continuous transitions**: • ferromagnetic transitions • liquid-vapor transitions in fluids, already observed in the XIX century





- first general framework was proposed by Landau (1937) $\rightarrow$ mean-field approx
- satisfactory understanding by the renormalization-group theory (Wilson 1971)

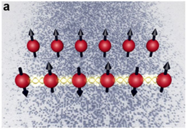
Simple interactions may give rise to complex phenomena, with long-distance correlations, after an appropriate tuning of the thermodynamic parameters.

Critical phenomena observed in many different materials have several features in common $\longrightarrow$ **Universality**.

• Ferromagnetism, Curie transition:
• magnetization from the spin of electrons in the imcomplete atomic shells of metal atoms: each electron carries one Bohr magneton
• interactions among spins due to exchange effects

$$H_{\rm spin} = -J \sum_{\langle ij \rangle} \vec{s}_i \cdot \vec{s}_j - \sum_i \vec{h} \cdot \vec{s}_i,$$

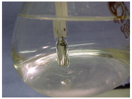
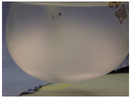
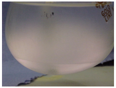
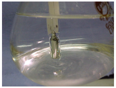
$$Z = \sum_{\{\vec{s}_i\}} e^{-H_{\rm spin}/T}, \qquad F = -\frac{T}{V} \ln Z$$



Ising model with $s_i = \pm 1$ describes uniaxial magnets

• Liquid-vapor transition with density instead of magnetization and chemical potential instead of magnetic field: $H_{\rm lattice\ gas} = -J \sum_{\langle ij \rangle} \rho_i \rho_j - \mu \sum_i \rho_i$, with $\rho_i = 0, 1$.

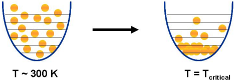
Ex.: critical opalescence in liquid systems, at their liquid-gas continuous transition, light diffusion when the correlation length increases from 10^{-9} m to 10^{-6} m

Bose-Einstein condensation in Bose gases

$$dN = \frac{d^3p dV}{(2\pi\hbar)^3} \frac{g}{e^{(\epsilon-\mu)/T} - 1}, \qquad T_c \longrightarrow \lambda_{DB} = \left(\frac{2\pi\hbar^2}{mT}\right)^{1/2} \approx d_{\rm atoms} = (N/V)^{-1/3}$$

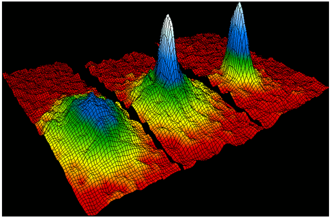
Below $T_c \approx \hbar^2 (N/V)^{2/3}/m$, a macroscopic number of atoms condenses to the lowest state, in a free gas $N_0/N = 1 - (T/T_c)^{3/2}$.



Interactions, even weak, give rise to a power-law critical behavior characterized by the U(1) symmetry of the condensate wave function.

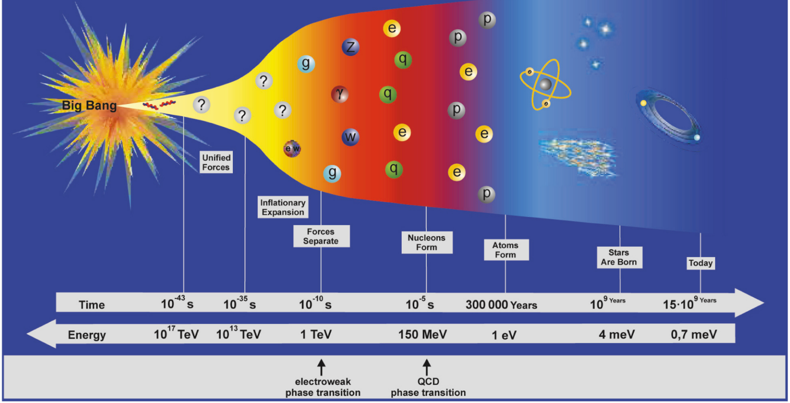
BEC recently observed in weakly interacting boson gases, made of alkali atoms, rubidium, sodium, lithium

velocity distribution of rubidium atoms



Earlier BEC-like phenomena in ^{4}He, which is a liquid rather than a gas, thus strongly interacting (only 9% atoms condense for $T \to 0$, providing the superfluid component)

Evolution of the Universe



Continuous transitions are characterized by power-law behaviors

- Disordered (symmetric) phase ($t \equiv T/T_c - 1 > 0,\ h = 0$):

$$\xi \sim t^{-\nu}, \quad C_H \sim t^{-\alpha}, \quad \chi \sim t^{-\gamma}, \quad \chi \sim \xi^{2-\eta}$$

- Ordered (broken) phase ($t < 0,\ h = 0^+$): $C_H \sim |t|^{-\alpha}$, $M \sim |t|^{\beta}$

- Critical isotherm ($t = 0,\ h > 0$): $\chi \sim |h|^{-\gamma/\beta\delta}$, $\widetilde{G}(q) \sim q^{-2+\eta}$

- Scaling equation of state: $h = t^{\beta\delta} F(z)$, $z = M t^{-\beta}$ in magnets (in fluids $h \to \rho - \rho_c$ and $M \to \mu - \mu_c$)

- Finite-size scaling, ex. $\chi \sim L^{2-\eta}$ at $t = 0$

- There are also critical behaviors characterized by exponential approaches: **LATTICE QCD** where $\xi \sim \exp(c\beta)$, and also 2D σ models, 2D KT transition

Many results for the **3D Ising universality class** characterized by a Z_2 parity symmetry (which may arise dynamically)

$\longrightarrow$ **quantum field theory** $\mathcal{L} = (\partial_\mu \varphi)^2 + r\varphi^2 + u\varphi^4$ with $\varphi \in \Re$

These global conditions apply liquid-vapor systems, fluid mixtures, uniaxial magnets, etc...

3D Ising exponents		ν	α	η	β
Experiments	liquid-vapour	0.6297(4)	0.111(1)	0.042(6)	0.324(2)
	fluid mixtures	0.6297(7)	0.111(2)	0.038(3)	0.327(3)
	uniaxial magnets	0.6300(17)	0.110(5)		0.325(2)
Theory PFT	6,7-l MZM	0.6304(13)	0.109(4)	0.034(3)	0.326(1)
	$O(\epsilon^5)$ exp	0.6290(25)	0.113(7)	0.036(5)	0.326(3)
Theory Lattice	HT exp	0.63012(16)	0.1096(5)	0.0364(2)	0.3265(1)
	MC	0.63020(12)	0.1094(4)	0.0368(2)	0.3267(1)
	MC	0.63002(10)			

Table: Estimates of the critical exponents of the 3D Ising universality class, from experiments, resummation of the FT 6,7-loop calculations within the MZM scheme and of $O(\epsilon^5)$ expansions, and from lattice techniques: 25th order high-temperature (HT) expansion and Monte Carlo (MC) simulations.

Main ideas to describe the critical behavior at a continuous transition

• **Order parameter** which effectively describes the critical modes

• **Scaling hypothesis**: singularities arise from the long-range correlations of the order parameter, diverging length scale

• **Universality**: the critical behavior is essentially determined by a few global properties: **the space dimensionality, the nature and the symmetry of the order parameter, the symmetry breaking**

RENORMALIZATION-GROUP THEORY

• RG flow in a Hamiltonian space

• the critical behavior is associated with a fixed point of the RG flow

• only a few perturbations are relevant, the corresponding positive eigenvalues are related to the critical exponents ν, η, etc...

The Gibbs free energy obeys a scaling law

$$\mathcal{F}_{\rm sing}(u_1, u_2, \dots, u_k, \dots) = b^{-d}\mathcal{F}_{\rm sing}(b^{y_1}u_1, b^{y_2}u_2, \dots, b^{y_k}u_k, \dots)$$

where u_i are the nonlinear scaling fields (analytic functions)

In a standard continuous transition there are **two relevant fields** u_i with $y_i > 0$, and an infinite set of irrelevant fields v_i with $y_i < 0$.

$$u_t \sim t, \quad u_h \sim H, \quad \text{for} \quad t, H \to 0$$

Setting $b^{y_t}|u_t| = 1 \to \mathcal{F}_{\rm sing} = |u_t|^{d/y_t}\mathcal{F}_{\rm sing}(u_h|u_t|^{-y_h/y_t}, v_i|u_t|^{-y_i/y_t})$
Since $v_i|u_t|^{-y_i/y_1} \to 0$ for $t \to 0$,

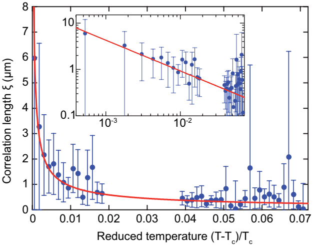
$$\mathcal{F}_{\rm sing} \approx |t|^{d/y_t} f(|H||t|^{-y_h/y_t}) + |t|^{d/y_t+\Delta_i} f_{(1,i)}(|H||t|^{-y_h/y_t}) + \dots$$

$y_t = 1/\nu$, $y_h = (\beta+\gamma)/\nu$, $\Delta_i = -y_i/y_t > 0$.

In the presence of other relevant perturbations beside t and H $\to$ multicritical behaviors

Phase transitions and critical phenomena in cold-atom experiments

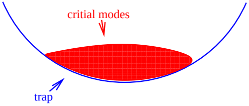
Finite-T transition related to the **Bose-Einstein condensation in interacting gases**, experiments show an increasing correlation length compatible with a continuous transition



Power-law critical behavior characterized by the U(1) symmetry of the condensate wave function, $\longrightarrow$ XY universality class, thus prediciting $\xi \sim |T - T_c|^{-\nu}$ with $\nu = 0.6717(1)$

The trapping potential confining the atoms significantly affects the critical behavior: correlations are not expected to develop a diverging length scale.

Trap-size scaling theory to describe how critical correlations develop in large traps.



From statistical models to quantum field theories

Ex.: **Ising model** defined on a d-dimensional lattice,
$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j, \quad \sigma_i = \pm 1, \quad Z = \sum_{\{\sigma_i\}} \exp(-H/T)$

- The critical behavior is due to the long-range modes, with $l \gg a$.

- As a result of a blocking procedure ($b \ll l$), preserving the symmetry,
$H_{\varphi^4} = \sum_{x,\mu} (\varphi_{x+\mu} - \varphi_x)^2 + u \sum_x (\varphi_x^2 - v^2)^2$, with $\varphi \in \Re$

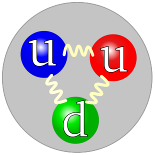
- The limit $a \to 0$ of H_{φ^4} should not change the universality class
$\mathcal{H}(\varphi) = \int d^d x \left[(\partial_\mu \varphi)^2 + r\varphi^2 + u\varphi^4 \right]$, $r - r_c \propto T - T_c$

- $Z = \int [d\varphi] \exp[-\mathcal{H}(\varphi)] \to \mathrm{QFT}$ with $\mathcal{H}(\varphi) \to \mathcal{L}(\varphi)$

- RG flow by a set of RG equations for the correlation functions

The way back provides a nonperturbative formulation of an Euclidean QFT, from the critical behavior of a statistical model.

Ex.: Nonperturbative formulation of the strong-interaction theory, from the critical regime of a 4D statistical system
explaining **the way from quarks to baryons**



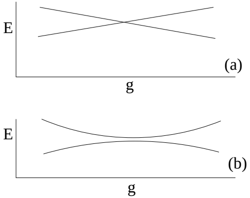
Quantum transitions in many-body systems

Phase transitions driven by quantum fluctuations, thus when $T \to 0$, $\to$ singular properties of the ground state and quantum critical behavior

Consider a quantum many-body theory described by the Hamiltonian $H = H_0 + gH_1$

- Level crossing if $[H_0, H_1] = 0$.
- More interestingly: avoided level crossing between the ground state and the first excited state, which closes approaching the infinite volume limit

leading to a nonanalicity at $g = g_c$



Nonanaliticity of the low-energy states, the gap $\Delta \to 0$ in the large-volume limit , the low-energy scale tend to zero

Quantum critical behavior with a peculiar interplay between quantum and thermal fluctuations at low T.

Continuous QPT $\longrightarrow$ diverging length scale ξ, and scaling properties, described by the RG scaling theory

Example: **the Ising chain in a transverse field**

$$H_{\rm Is} = -J\sum_i \sigma_i^x \sigma_{i+1}^x - g\sigma_i^z\,, \quad \sigma_i = \text{Pauli matrices}$$

$g = 0 \longrightarrow$ two degenerate ground states $\prod_i |\rightarrow_i\rangle$ and $\prod_i |\leftarrow_i\rangle$

$g \to \infty \longrightarrow$ GS=$\prod_i |\uparrow_i\rangle$, breaking Z_2

These phases extend to finite g, quantum transition at $g_c/J = 1$, between quantum paramagnetic and ordered phases

Continuous QPT $\longrightarrow$ diverging length scale ξ, and scaling properties.

2D Ising quantum critical behavior with $\Delta \sim \xi^{-1} \sim |g - g_c|$

Universality properties and renormalization-group theory extend to quantum transitions

A QPT is generally characterized by a relevant parameter g, with RG dimensions $y_g \equiv 1/\nu$

$$\xi \sim |\bar{g}|^{-\nu}, \qquad \Delta \sim |\bar{g}|^{z\nu} \sim \xi^{-z}, \qquad \bar{g} = g - g_c$$

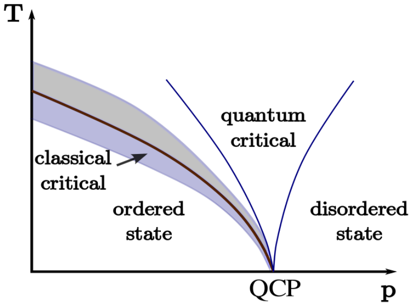
The temperature represents a *relevant* parameter, characterized by its power law, i.e. $\xi \sim T^{-1/z}$ at the critical point.

Scaling law of the free energy $F(\mu, T) = b^{-(d+z)} F(\bar{g} b^{1/\nu}, T b^z)$

Typical phase diagrams $\longrightarrow$

- classical description at the finite-T transition line, when $\hbar\omega_{\rm crit} < k_B T$

- Quantum scaling laws describe the critical behavior around the QCP, arising from the interplay between thermal and quantum fluctuations



Like *classical* transitions, global features, such as the summetry and symmetry breaking pattern, determine the critical behavior

Ex.: The quantum critical behavior of the d-dim quantum Ising/Heisenberg models shares the same universal features of the $(d+1)$-dim classical Ising/Heisenberg models, such as the critical exponents, etc...

Reflecting the quantum-to-classical mapping from d-dim quantum many-body systems to $(d+1)$-dim classical statistical systems

Several applications:

- Quantum magnetism and criticality
- Magnetic excitations of the insulator $LiHoF_4$, quantum Ising transition
- Quantum Heisenberg antiferromagnets, the insulator La_2CuO_4
- High-T superconductors
- Quantum particle systems
- BCS to BEC transition in Fermi atomic systems
- new matter states, such as spin liquids, disorder down to very low T

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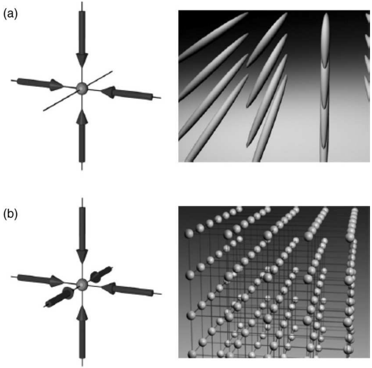
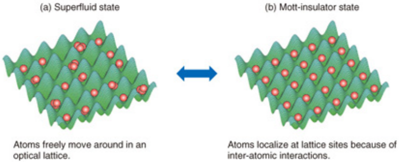
Quantum phase transitions in cold-atom experiments





Ex.: **Quantum Mott insulator to superfluid transitions** and different Mott phases (where the density is independent of μ)

A common feature is a **confining potential**, which can be varied to achieve different spatial geometries, allowing also to effectively reduce the spatial dims